## The Time Value of Money Equation

The time value of money equation

$$0 = PV + (1 + ip)\,PMT\left[\frac{1 - (1 + i)^{-N}}{i}\right] + FV\,(1 + i)^{-N}$$

establishes the relationships between the following variables:

| | |
|---|---|
| *N* | The number of monthly payments or compounding periods. |
| *I%YR* | The annual interest rate as a fraction ($i = I\%YR \div 1200$). |
| *PV* | The present value. (This can also be an initial cash flow or a discounted value of a series of future cash flows.) *PV* always occurs at the beginning of the first month. |
| *PMT* | The monthly payment. |
| *FV* | The future value. (This can also be a final cash flow or a compounded value of a series of cash flows.) *FV* always occurs at the end of the *N*th month. |

The value *p* indicates payment timing. If $p = 1$, then payments occur at the *beginning* of each month. If $p = 0$, then payments occur at the *end* of each month. The "TVM" program uses flag 00 to represent *p*. For payments at the beginning a each month, set flag 00. For payments at the end of each month, clear flag 00.

**Note**

The correct *sign* (positive or negative) for TVM numbers is essential. The calculations will make sense only if you consistently show *payments out* as *negative* and *payments in* (receipts) as *positive*. Perform a calculation from the point of view of *either* the lender (investor) *or* the borrower, but not both!



Here is how the equation can be written as a program for the Solver:

```
01 LBL "TVM"
02 MVAR "N"
03 MVAR "I%YR"
04 MVAR "PV"
05 MVAR "PMT"
06 MVAR "FV"
```

Declares the menu variables.

```
07 1
08 ENTER
09 ENTER
10 RCL "I%YR"
11 %
12 12
13 ÷
```

Calculates the monthly interest rate expressed as a decimal fraction, *i*.

```
14 STO ST T
```

X = *i*, Y = 1, Z = 1, T = *i*

```
15 FC? 00
16 CLX
17 +
```

If flag 00 is clear (End mode), calculates ($i + 0$). If flag 00 is set (Begin mode), calculates ($i + 1$).

```
18 R↓
19 +
20 RCL "N"
21 +/-
22 Y↑X
```

Calculates $(1 + i)^{-N}$.

```
23 1
24 X<>Y
25 -
```

Calculates $1 - (1 + i)^{-N}$.

```
26 LASTX
27 RCL× "FV"
```

Calculates $FV\,(1 + i)^{-N}$.

```
28 R↓
29 X<>Y
30 ÷
31 ×
```

Calculates $(1+ip)\left[\frac{1 - (1 + i)^{-N}}{i}\right]$.

```
32 RCL× "PMT"
33 +
34 RCL+ "PV"
35 END
```

**Example.** Penny of Penny's Accounting wants to know what the monthly payments will be for a 3-year loan at 10.5% annual interest, compounded monthly. The amount financed is $5,750. Payments are made at the end of each period.

After keying in the program above, use the Solver to calculate the unknown information for Penny.

SOLVER TVM


Clear flag 00 and set the display format to FIX 2.

FLAGS CF 00
DISP FIX 02


Enter the known values: $PV = 5750$, $FV = 0$, $I\%YR = 10.5$, and $N = 3 \times 12$.

5750 PV


0 FV


10.5 I%YR


3 ENTER 12 × N


Now solve for the payment.

PMT


The payment is negative because it is money to be *paid out*.

This is $10 higher than Penny's client can pay each month. What interest rate would reduce the monthly payments by $10? *Add* 10 to the negative payment that's already in the *X*-register and store the new value into *PMT*.



10 + PMT


Now, solve for the interest rate.

I%YR


Return to FIX 4 display mode and exit from the Solver.

DISP FIX 04 EXIT EXIT