Commodore Business Machines, Inc.
901 California Avenue
Palo Alto, California 94304, USA

Commodore/MOS Technology
Valley Forge Corporate Center
950 Rittenhouse Road
Norristown, Pennsylvania 19401, USA

Commodore Business Machines Limited
3370 Pharmacy Avenue
Agincourt, Ontario, Canada M1W2K4

Commodore Business Machines Limited
Eaglescliffe Industrial Estate
Eaglescliffe, Stockton on Tees
Teeside TS 160 PN, England

Commodore Büromaschinen GmbH
Frankfurter Strasse 171-175
6078 Neu Isenburg
West Germany

Commodore Japan Limited
Taisei-Denshi Building
8-14 Ikue 1-Chome
Asahi-Ku, Osaka 535, Japan

Commodore Electronics (Hong Kong) Ltd.
Watsons Estates
Block C, 11th floor
Hong Kong, Hong Kong

commodore

PRINTED IN HONG KONG

P50 Programmable Calculator

Owner's Manual

Keyboard Index

Introduction

The Commodore P50 Programmable calculator offers a wide variety of mathematical operations and the power of a 24 step programming capability, all at a very low cost.

The single 3/4" x 1/2" x 1/16" microprocessor chip is the heart and brains of your new calculator. It is unique; virtually no other calculator packs as much power in a single chip. This accounts for the remarkable cost efficiency. The chip is a product of the superb engineering and production skills of MOS Technology, a Commodore company.

This chip contains enough circuitry to generate trigonometric, inverse trigonometric, logarithm and exponential functions. In addition to the square, square root, reciprocal and factorial operators, there is a useful integer function which truncates the decimal part of a number. Five memory operators simplify computations with the single memory, and thus reduce the number of steps needed in many programs.

The full potential of the P50 is realized in the programming feature that allows up to 24 keystrokes to be stored in the machine. Loops can be formed using the GOTO key, thus enabling thousands of operations to be performed at the touch of a button. There is conditional branching on positive and negative numbers as well as zero.

This manual is designed to familiarize the reader step by step with the P50. There are sample problems that reinforce this procedure and a list of 16 useful programs including polar/rectangular coordinate conversion, degrees/dms conversion, binary numbers, the quadratic formula, Fibonacci numbers, compound interest, loans and dice.

We at Commodore take great pride in this calculator. We feel that exciting new applications in mathematics and programming will be opened up to you, the new owner.

Table of Contents

I. Preliminaries

Power On

Your programmable calculator can run on battery power alone or you can use the optional AC adapter.

Push the ON/OFF slide switch to the left to turn the calculator on. A gold dot appears to the right of the switch and the display should read 0.

The Display

A sample display is shown above. The number on the display reads

3141.5927×10^{-3}

Your calculator can compute numbers as large as

99999999×10^{99}

and as infinitesimal as

$.00000001 \times 10^{-99}$

Display Shut-Off Feature

To conserve battery power, the display has a timed shut-off feature. After 60 seconds of non-use, the displayed digits will disappear leaving only the decimal point. No information is lost and the calculations can continue at any time. Press [x↔y] **twice to recall the number to the display.**

Entry

Enter numbers exactly as they appear, using the digit keys and the decimal key [•]. To enter a negative number, press the change sign key [+/−].

The [+/-] key will also change a negative number on the display to a positive one.

Scientific Form

Scientists usually express numbers in the following way:

6.023×10^{23}

This is called *scientific form* and can be easily entered with the following steps.

(1) Enter mantissa, **6.023**

(2) If the number is negative, press [+/-]

(3) Enter exponent by pressing [EE] **23**

(4) If exponent is negative, press [+/-]

The Clear Key [C/CE]

If you make a mistake on entry, press [C/CE] once. This will clear only the display and will leave the information stored in the registers.

To clear a continuing calculation, press [C/CE] twice.

The Pi Key [π]

Press [π] to display

3.1415927

II. Arithmetic Functions

The simple arithmetic keys [+] [–] [x] [÷] [=] are used to perform simple arithmetic exactly as written. For example, to find 3 x 4 =, just press **3** [x] **4** [=] and the answer, **12**, appears on the display.

Any of the keys [+] [–] [x] [÷] can be overridden by another. If you make a mistake, just reenter the correct one. For example,

3 [+] [x] **4** [=] will ignore the [+] and display the answer as **12**.

Chaining: Two examples of chained operations are

3 x 4 + 5 =   8 – 4 ÷ 2 =

According to the rules of algebra, x and ÷ supersede + and –. So we should get

3 x 4 + 5 = (3 x 4) + 5   8 – 4 ÷ 2 = 8 – (4 ÷ 2)
= 12 + 5   = 8 – 2
= 17   = 6

This is not the case on this calculator (and on most calculators with algebraic logic). Each time an arithmetic key is pressed, the preceding operation is performed and the result is displayed. Thus, **8** [–] **4** [÷] will display **4**, the answer to 8 – 4. When you push **2** [=], the calculator will perform **4** ÷ **2** and display **2**. Thus, the above operations on the calculator yield

3 [x] **4** [+] **5** [=] ⟶ **17**
8 [–] **4** [÷] **2** [=] ⟶ **2**

Chaining is useful because complex expressions like

(((3 + 2) ÷ 11) x 44) – 6 =

can be calculated without using the memory.

3 [+] 2 [÷] 11 [x] 44 [–] 6 [=] ⟶ 14

On the other hand, the following expression must be rearranged before calculation.

3 + (6 x ((2 x 12) ÷ 8)) =
(((2 x 12) ÷ 8) x 6) + 3

2 [x] **12** [÷] **8** [x] **6** [+] **3** [=] ⟶ **21**

Practice Problems: Compute

$\left(\frac{\pi}{4} + 1.7\right) - 4.623$ $3.3 + \frac{6.5 - 1.42}{\pi}$

Answers: –2.1376018   4.9170142

With a little practice, you will find that you can chain complex operations without having to rewrite the expression on paper. You will quickly find that x and + give you no trouble, but – and ÷ present some problems. Try to calculate

2 – (4 ÷ (3 + 1))

It is for this reason we have the following key.

The Exchange [x↔y] **Key:** In binary operations ([+] [–] [x] [÷]) there are two numbers stored in the registers. For example, after pressing 3 [+] 4, the y register contains 3 and the x register (the display) contains 4. When you press [x↔y] the registers are switched.

You can use this feature to check a number already entered. For example, the number 6.626×10^{-27} is in the display. You press [÷] 9 but then remember that you should have written the previous number down. Simply press [x↔y] and the display reads **6.626 x** $\mathbf{10^{-27}}$. You write this down, restore the registers by pressing [x↔y] again and continue.

The major use of the [x↔y] key, however, is in chaining. Now expressions like

2 – (4 ÷ (3 + 1))

can be calculated without using the memory.

3 [+] 1 [÷] 4 [x↔y] [–] 2 [x↔y] [=] ⟶ 1.

Practice Problems: Compute

$\pi - \dfrac{\pi}{\pi - 1}$ 6.23 – (4.41 – (3.62 – 1.7))

Answer: 1.6746504 , 3.74

Algebraic Operators include:

[x^2] **Square Key:** Press [x^2] to square the displayed number.

[$\sqrt{x}$] **Square Root Key:** Press [$\sqrt{x}$] to take the square root of the displayed number.

[1/x] **Reciprocal Key:** Press [1/x] for the reciprocal of the displayed number.

Example: Find

$$w = \sqrt{\left(\frac{1}{x}\right)^2 + \left(\frac{1}{y}\right)^2 + \left(\frac{1}{z}\right)^2}$$

where $(x, y, z) = (3, 4, 5)$

Solution:

3 [1/x] [x^2] [+]
4 [1/x] [x^2] [+]
5 [1/x] [x^2] [=]
[$\sqrt{x}$] ⟶ 0.4621808

Practice Problems: Compute

$\dfrac{1}{\sqrt{\pi - 1}}$ $\pi^2 + \pi^4 + \pi^8$

Answer: 0.6833317   9595.8097

Factorial [n!] is defined by

n! = n x (n – 1) x (n – 2) x · · · x 2 x 1

for any positive integer n. (0! = 1).. For example,

4! = 4 x 3 x 2 x 1 = 24

Press [n!] to compute n! for a displayed non-negative integer where n is less than 70.

Example: Compute $\dfrac{11!}{7!4!}$

Solution:

7 [n!] [x]
4 [n!] [÷]
11 [n!] [x↔y]
[=] ⟶ 330

Practice Problem: Compute

$$\frac{1}{6!}+\frac{1}{7!}+\frac{1}{8!}+\frac{1}{9!}$$

Answer: 1.6148589×10^{-3}

The Integer Function [INT] will truncate (drop off) the decimal part of the displayed number. For example, **3.1415** [INT] will drop the .1415 and display 3. Similarly, **−55.999** [INT] will display −55. This key will be very useful in programming (see Appendix A12).

Example: Compute int (π^4)

Solution: [π] [x^2] [x^2] [INT] ⟶ **97**

Practice Problem: Compute

$$\left\{\text{int}\left(\frac{\text{int}\sqrt{5151}}{\text{int}\sqrt{151}}\right)\right\}!$$

Solution: 120

III. Trigonometric Operators

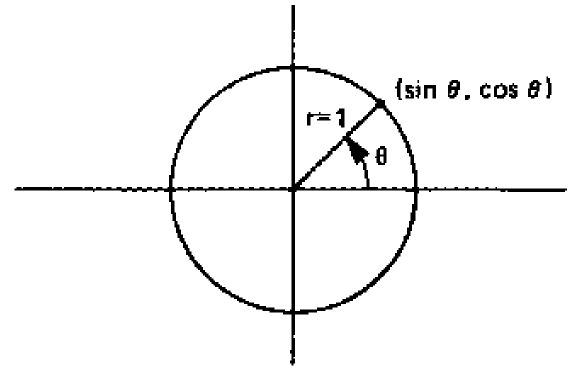
The [DEG] [RAD] [GRAD] keys represent the three units of measurement for angles:

1 circle = 360 degrees
= 2π radians
= 400 gradians

Before using the trig keys, you must put the calculator in the right *angle mode*. That is, you must choose whether you want your entries and answers to be expressed in degrees, radians or gradians.

The machine is naturally operating in degree mode. Press [RAD] to enter radian mode and [GRAD] to enter gradian mode. Press [DEG] to return to degree mode.

The trig functions sin θ and **cos θ** are defined in the following diagram.

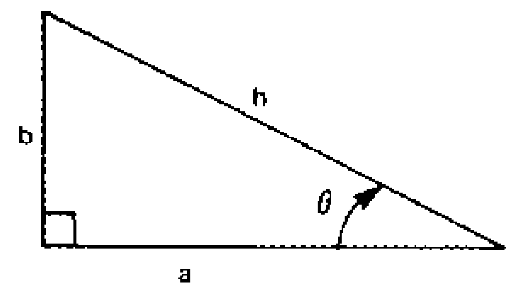


where (sin θ, cos θ) are the rectangular coordinates of the indicated point.

The tangent is defined as

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

The trig functions have the property that if



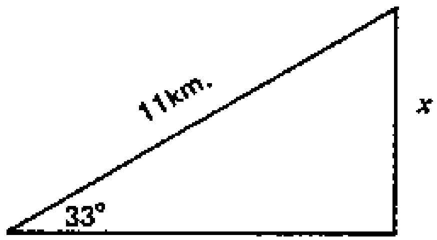
$$\text{then } \sin\theta = \frac{b}{h} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{a}{h} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{b}{a} = \frac{\text{opposite}}{\text{adjacent}}$$

The trig keys [sin] [cos] [tan] instantly compute the sine, cosine and tangent of the angle displayed. Remember to put the calculator in the appropriate angle mode using the [DEG] [RAD] [GRAD] keys as explained in the previous section.

Example: Find x



Solution:

$$\sin 33° = \frac{x}{11\text{ km}}$$

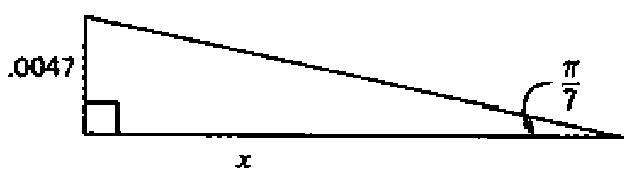
Therefore, $x = (11 \times \sin 33°)$ km.

The program is

33 [sin] [x] 11 [=] ⟶ 5.9910294

Therefore, $x = 5.991$ km.

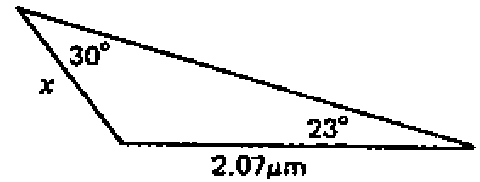
Practice Problem: Find x



Hint: [RAD], [x↔y] and [=] are important keys in this computation.

Answer: 9.7596506×10^{-3}

Example: Find x



Solution: Use the law of sines (Appendix B1)

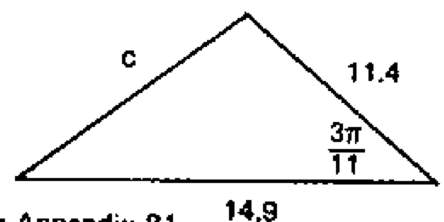
$$\frac{x}{\sin 23°} = \frac{2.07}{\sin 30°}$$

Hence, $x = \frac{2.07}{\sin 30°} \times \sin 23°$

2.07 [÷]
30 [sin] [x]
23 [sin] [=] ⟶ 1.6176269

Thus $x = 1.618\ \mu m$.

Practice Problem: Find c



Hint: Appendix B1

Answer: 11.379838

The inverse trig functions are the reverse of the trig functions. The trig functions take an angle θ and give you a number x. The inverse trig functions take a number x and give you an angle θ.

The inverse sine, cosine and tangent are denoted

arcsine, arccosine, arctangent

and are defined by

arcsine $x = \theta \Leftrightarrow \sin\theta = x$ $(-180° \le \theta \le 180°)$
arccosine $x = \theta \Leftrightarrow \cos\theta = x$ $(0 \le \theta \le 180°)$
arctangent $x = \theta \Leftrightarrow \tan\theta = x$ $(0 < x)$

Inverse functions do the reverse operations of their associated functions. Thus we have

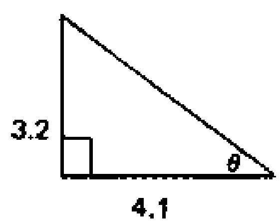
arcsine $(\sin\theta) = \theta$	$\sin(\text{arcsine } x) = x$
arccosine $(\cos\theta) = \theta$	$\cos(\text{arccosine } x) = x$
arctangent $(\tan\theta) = \theta$	$\tan(\text{arctangent } x) = x$

whenever θ and x fall within the above constraints.

To take the arcsine of the number on display, press [arc] [sin]. The answer is an angle expressed in degrees, radians or gradians as indicated by the angle mode (pg 10).

Similarly, to calculate arccosine and arctangent, press [arc] [cos] and [arc] [tan].

Example: Find θ in gradians.



Solution: We have $\tan\theta = \frac{3.2}{4.1}$

Therefore,

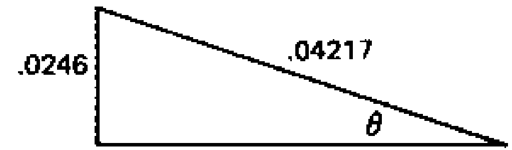
$$\theta = \arctan\frac{3.2}{4.1}$$

Thus,

[GRAD]
3.2 [÷] 4.1 [=]
[arc] [tan] ⟶ 42.190671

So, $\theta = 42.19$ gradians

Practice Problems: Find θ in degrees



Answer: 35.686729°

Find θ in degrees

$$\theta = \text{arccosine}\left(\sin\frac{6\pi}{19}\right)$$

Answer: 33.157895°

IV. Transcendental Operators

[ln] **Natural Log Key:** This key computes the natural log (ln) of the displayed number.

[e^x] **Natural Antilog Key:** This key computes e^x for a displayed number x.

[log] **Log Key:** This key computes the log to the base 10 of the displayed number.

[10^x] **Antilogarithm Key:** This key computes the antilog of the displayed number.

Properties of Transcendental Functions:

e^x and ln have the following properties:

(i) $\ln a + \ln b = \ln(a \times b)$
(ii) $\ln a - \ln b = \ln(a \div b)$
(iii) $b \ln a = \ln(a^b)$
(iv) $e^{\ln x} = x$
(v) $\ln e^x = x$

10^x **and log** have the following similar properties:

(vi) $\log a + \log b = \log(a \times b)$ (ix) $10^{\log x} = x$
(vii) $\log a - \log b = \log(a \div b)$ (x) $\log 10^x = x$
(viii) $b \log a = \log(a^b)$

Example: A colony of bacteria has the following population formula:

$$n = 3.6 \times 10^{2t} + 4.9 \times 10^4$$

Here, the number of organisms, n, is determined by the number of days, t. How long will it take the population to reach 100 million?

Solution: Solve for t

$$3.6 \times 10^{2t} + 4.9 \times 10^4 = 10^8$$
$$\Rightarrow 3.6 \times 10^{2t} = 10^8 - (4.9 \times 10^4)$$
$$\Rightarrow 10^{2t} = \frac{10^8 - (4.9 \times 10^4)}{3.6}$$

Take the log of both sides

$$\log 10^{2t} = \log\left(\frac{10^8 - (4.9 \times 10^4)}{3.6}\right)$$

By property (x) above,

$$\log 10^{2t} = 2t$$

Therefore,

$$t = \frac{1}{2}\log\left(\frac{10^8 - (4.9 \times 10^4)}{3.6}\right)$$

Now, compute t

1 [EE] 8 [–]
4.9 [EE] 4 [÷]
3.6 [=]
[log]
[÷] 2 [=] ⟶ 3.7217423
[–] 3 [x] 24 [=] ⟶ 17.321816

Thus it will take approximately 3 days and 17 hours to reach a population of 100 million.

Practice Problem: Calculate

$$e^{(\pi + e^{\pi})}$$

Answer: 2.5956819×10^{11}

V. The Memory

[STO] **Store Key:** stores the displayed number in the memory. This will override the previous entry in the memory.

[RCL] **Recall Key:** displays the contents of the memory.

[M+] **Add to Memory Key:** adds the displayed number to the number stored in the memory. The result is then stored in the memory.

[Mx] **Multiply by Memory Key:** multiplies the displayed number by the number stored in the memory. The result is then stored in the memory.

[x↔M] **Memory Exchange Key:** displays the contents of the memory and at the same time stores the displayed number in the memory.

Example: Find

$$\frac{e^x - e^{-x}}{2} \quad \text{if} \quad x = \sqrt{\frac{1}{\sin^2 11^\circ} + \frac{1}{\cos^2 23^\circ}}$$

Solution:

11 [sin] [x^2] [1/x] [+]
23 [cos] [x^2] [1/x] [=]
[$\sqrt{x}$] [STO]
[RCL] [ex] [–]
[RCL] [+/–] [ex] [÷]
2 [=] ⟶ 105.53932

Practice Problem: Compute

$$\left(\frac{1}{3!}\right)^2 + \left(\frac{1}{4!}\right)^2 + \left(\frac{1}{5!}\right)^2 + \left(\frac{1}{6!}\right)^2 + \left(\frac{1}{7!}\right)^2$$

Answer: 2.9585302×10^{-2}

Practice Problem: Compute

$$\frac{e^\pi + \pi}{e^\pi - \pi}$$

Answer: 1.3141734

Example: Compute z

$$z = 3x^4 + x^3 - 2x^2 + 1 \quad \text{if} \quad x = \sqrt{\pi} - 1$$

Solution: This formula cannot be computed directly. A simple trick is to rearrange the expression:

$$z = (3x^2 + x - 2)x^2 + 1$$
$$= ((3x + 1)x - 2)x^2 + 1$$

Now compute z:

π [$\sqrt{x}$] [−] 1 [=] [STO]
3 [×] [RCL] [+] 1
[×] [RCL] [−] 2
[×] [RCL] [x^2]
[+] 1 [=] ——→ 1.3356405

Practice Problem: Compute z

$$z = 3x^{14} + 2x^{10} - 1 \quad \text{if } x = \ln \pi$$

Hint: $x^{10} = ((x^2)^2)^2 \times x^2$

Answer: 26.63294

When writing programs, it will be useful to keep track of the display and memory as shown in the next example.

Example: Compute

$$A = \sum_{i=1}^{5} \pi^i = \pi + \pi^2 + \pi^3 + \pi^4 + \pi^5$$

Solution:

Enter	x*	M
π	π	0
[STO]	π	π
[Mx]	π	π^2
[M+]	π	$\pi^2 + \pi$
[Mx]	π	$\pi^3 + \pi^2$
[M+]	π	$\pi^3 + \pi^2 + \pi$
[Mx]	π	$\pi^4 + \pi^3 + \pi^2$
[M+]	π	$\pi^4 + \pi^3 + \pi^2 + \pi$
[Mx]	π	$\pi^5 + \pi^4 + \pi^3 + \pi^2$
[M+]	π	$\pi^5 + \pi^4 + \pi^3 + \pi^2 + \pi$
[RCL] ——→	447.44625	

*The Display

Practice Problem: Compute

$$A = \sum_{i=4}^{9} x^i \quad \text{where } x = \sqrt{\frac{9!}{4!}}$$

Answer: 6.4793195×10^{18}

VI. The Programming Keys

[LRN] **The Learn Key:** is used to enter a program into the calculator. Press [LRN] before entering the program. The display will read **00**. This indicates step 00 is the next step to be entered. As you enter keystrokes, the display reads the step number fo the next step. After entering the program, press [LRN] to return to compute mode.

[R/S] **The RUN/STOP Key:** has two functions.

In the learn mode, press [R/S] for **STOP**. When executing a program, the calculator will stop at this point. You can then read a result or enter some data or both.

In the compute mode, press [R/S] for **RUN**. The machine continues in the program where it left off.

[GOTO] **The GOTO Key:** has two functions.

In the learn mode, press [GOTO] **09** and the machine will go to step 9 whenever it encounters this step in the program. This is used to form loops.

In the compute mode, use [GOTO] before the [R/S] (**RUN**) key to tell the machine where to start the computations. For example, you usually press [GOTO] **00** before executing a new program.

[SKZ] [SKN] [SKP] **The Conditional Branching Keys:** are used only in the learn mode.

The [SKZ] key means skip if zero. When the machine encounters [SKZ] in the execution of a program, it checks the current displayed number. If this is zero, it skips the next step. (If the next step is [GOTO], it skips two steps; i.e., [GOTO] 04 is two steps.) If the number is not zero, it just continues.

Similarly, [SKN] and [SKP] test for a negative sign on the displayed number. Thus, [SKN] means skip if negative and [SKP] means skip if positive or zero.

[SSTP] **The Single Step Key:** is helpful when debugging programs. In the compute mode, [SSTP] is the same as **RUN** but will only execute one step. By repeatedly pressing [SSTP], you get to see each intermediate calculation in the program.

Note: There are 24 steps available, numbered **00** to **23**. Each key entry takes up a single step; e.g., 329 takes three steps.

There are two exceptions:

[ARC] takes 0 steps: [ARC] [sin] is one step.

After [GOTO], the next two digits take one step: [GOTO] **04** are two steps.

At this time you, the reader, should attempt some of the programs in Appendix A. This will familiarize you with the format we use and will give you practice entering and executing programs.

In the next chapter, we analyze some sample problems and describe the techniques for writing your own programs.

VII. Programming

Evaluating Functions

Example: Find $f(x)$ for x = .9,51,99999 and 0.0101.

$f(x) = \ln x - (e^{-\arctan x}) + 9.4$

Solution: Find the keystroke sequence for this function using RCL for x:

RCL ln – RCL arc tan +/– e^x + 9.4 =

Next, compile the program:

PROGRAM

LRN

00	STO	08	9
01	ln	09	.
02	–	10	4
03	RCL	11	=
04	arctan	12	STOP
05	+/–	13	GOTO
06	e^x	14	00
07	+		

LRN

EXECUTE

GOTO 00
Enter x
RUN → f(x)
Enter x
RUN → f(x)
.
.
.

Now, run the program for the given values.

GOTO 00
Enter .9
RUN ——→ 9.2946395
Enter 51
RUN ——→ 13.331826
Enter 99999
RUN ——→ 20.912915
Enter .0101
RUN ——→ 4.2441353

Practice Problem: Find f(x) for

x = 1, 2, 9, 0.01102 and -1.09×10^{-23}

if $f(x) = \sqrt{\ln(e^x + e^{-x})}$

Hint: Don't forget the [=]

Answer: f(1) = 1.0615687
f(2) = 1.420616
f(9) = 3
f(0.01102) = .832591
f(-1.09×10^{-23}) = .8325546

Plotting Curves

Example: Plot

$f(x) = \sin\left(\frac{1}{x}\right)^\circ$ for $0.001 \leq x \leq 0.020$

using increments of .001.

Solution: The formula for f(x) is

RCL $\frac{1}{x}$ sin

We will write the program to increment the memory by .001; evaluate f(x); stop and then repeat. The program is

PROGRAM

LRN

00	RCL	07	STO
01	+	08	1/x
02	.	09	sin
03	0	10	STOP
04	0	11	GOTO
05	1	12	00
06	=		

LRN

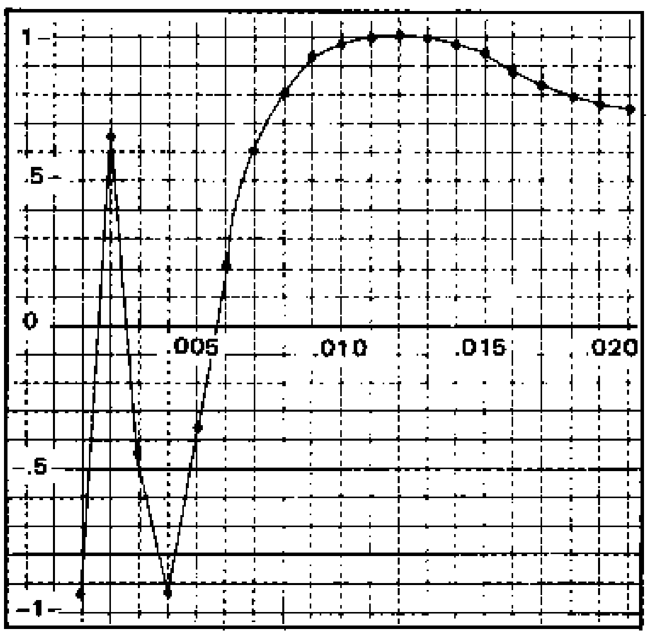
EXECUTE

GOTO 00
RUN ——→ f(.001)
RUN ——→ f(.002)
RUN ——→ f(.003)
·
·
·

We get (rounded to 3 decimal places):

f(.001) = –.985	f(.011) = 1
f(.002) = .643	f(.012) = .993
f(.003) = –.449	f(.013) = .974
f(.004) = –.940	f(.014) = .948
f(.005) = –.342	f(.015) = .918
f(.006) = .231	f(.016) = .887
f(.007) = .604	f(.017) = .856
f(.008) = .819	f(.018) = .826
f(.009) = .933	f(.019) = .795
f(.010) = .985	f(.020) = .766

If we plot this on a graph, we get

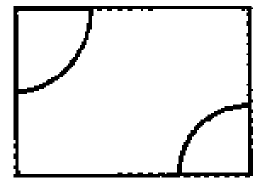


Practice Problem: Plot

f(x) = tan (e^x + 44)°

for 3.0 ≤ x ≤ 4.5
using increments of .1

Answer:



Polynomials

Example: Find p(x) for $x = 4, 7, 6, \frac{1}{2}, \frac{1}{3}$ where

$p(x) = 3x^4 + 2x^3 - x + 1$

Solution: As shown on page 18 you cannot enter this formula directly. You should first rearrange the expression

$$\begin{aligned} p(x) &= [3x^4 + 2x^3 - x] + 1 \\ &= (3x^3 + 2x^2 - 1)\,x + 1 \\ &= ((3x + 2)\,x^2 - 1)\,x + 1 \end{aligned}$$

Thus, the key sequence for p(x) is

RCL x 3 + 2 x RCL x^2
– 1 x RCL + 1 =

Next, compile the program:

PROGRAM

LRN

00	STO	09	1
01	x	10	x
02	3	11	RCL
03	+	12	+
04	2	13	1
05	x	14	=
06	RCL	15	STOP
07	x^2	16	GOTO
08	–	17	00

LRN

EXECUTE

```
GOTO 00
Enter  x
RUN ——→ p (x)
Enter  x
RUN ——→ p (x)
  ⋮
```

Now compute the results

```
GOTO 00
Enter   4
RUN ——→ 893
Enter   7
RUN ——→ 7883
Enter   6
RUN ——→ 4315
Enter 1 ÷ 2 =
RUN ——→ .9375
Enter 1 ÷ 3 =
RUN ——→ .7777777
```

Thus, p(4) = 893 $p\left(\frac{1}{2}\right)$ = .9375

p(7) = 7883 $p\left(\frac{1}{3}\right)$ = .7778

p(6) = 4315

Practice Problem: Find

$p(x)$ for x = .17, .84, 3.6, 19

if $p(x) = x^4 - 2x^3 + x^2 - x - 1$

Answer: −1.1500908, −1.8219366, 83.0096 , 116,944

Roots of Polynomials

A root of a polynomial is a number x^* such that

$p(x^*) = 0$

Suppose you are given a polynomial. For example,

$p(x) = 15x^3 - 34x^2 + 4x + 8$

The divisors of the leading coefficient, 15 are

15, 5, 3, 1

The divisors of the constant, 8 are

8, 4, 2, 1

Make a list of all fractions $\frac{a}{b}$ where a divides evenly into the constant and b divides evenly into the leading coefficient.

$\frac{8}{1}\ \frac{4}{1}\ \frac{2}{1}\ \frac{1}{1}\ \frac{8}{3}\ \frac{4}{3}\ \frac{2}{3}\ \frac{1}{3}$

$\frac{8}{5}\ \frac{4}{5}\ \frac{2}{5}\ \frac{1}{5}\ \frac{8}{15}\ \frac{4}{15}\ \frac{2}{15}\ \frac{1}{15}$

Evaluate $p(x)$ for each of the above fractions and their negatives. If $p(x) = 0$ then x is a root of $p(x)$.

(Due to rounding errors, a number as small as 10^{-8} should be considered equal to zero.)

This technique will find all the rational (fractional) roots of any polynomial. This cannot be used for irrational roots ($\sqrt{2}$, $\sqrt{3}$, etc.).

Practice Problems: Find the roots of $p(x)$ above

Answer: 2, $\frac{2}{3}$, $-\frac{2}{5}$

Find the roots of

$p(x) = 2x^3 - 13x^2 + x + 70$

Answer: 5, $\frac{7}{2}$, −2

The [x↔y] Key

The [x↔y] is useful in many programs as shown in the next example.

Example: Find f(x) for x = 1, 5, 10, 100

$$f(x) = \frac{e^x}{\ln x - \sqrt{x+4}}$$

Solution: First write out the key sequence for the formula:

RCL + 4 = $\sqrt{x}$ − RCL
ln $x \leftrightarrow y$ ÷ RCL e^x
$x \leftrightarrow y$ =

Notice how $x \leftrightarrow y$ is used in this example. Next, compile the program.

PROGRAM

LRN			
00	STO	09	÷
01	+	10	RCL
02	4	11	e^x
03	=	12	$x \leftrightarrow y$
04	$\sqrt{x}$	13	=
05	−	14	STOP
06	RCL	15	GOTO
07	ln	16	00
08	$x \leftrightarrow y$		
LRN			

EXECUTE

GOTO 00
Enter x
RUN ⟶ f(x)
Enter x
RUN ⟶ f(x)
.
.
.

Run the program

GOTO 00
Enter 1
RUN ⟶ −1.2156526
Enter 5
RUN ⟶ −106.7289
Enter 10
RUN ⟶ −15306.018
Enter 100
RUN ⟶ $-4.8063297 \times 10^{42}$

Practice Problem: Find f(θ) for

$\theta = \frac{11\pi}{19}$ radians
$\theta = 46°$
$\theta = 90$ gradians

where $f(\theta) = \tan\theta - \sqrt{\sin\theta + \cos\theta}$

Answer: $f\left(\frac{11\pi}{19}\right) = -4.7997425$
f(46°) = −.1535862
f(90 grad) = 5.244114

The $x \leftrightarrow M$ Key

The $x \leftrightarrow M$ key is useful in many programs as shown in the following example.

Example: Find

$$\sum_{i=1}^{n} \sqrt{i} = \sqrt{1} + \sqrt{2} + \sqrt{3} + \cdots + \sqrt{n}$$

for n = 5, 11, 19, 104

Solution: Suppose, for the time being, n = 11. It is easier to find

$$\sqrt{11} + \sqrt{10} + \sqrt{9} + \cdots + \sqrt{1}$$

Set up a table and experiment with different arrangements:

Enter	x	M	x	M	x	M
11	11	0				
$x \leftrightarrow M$	0	11	$\sqrt{11}$	10	$\sqrt{11}+\sqrt{10}$	9
+	0	11	$\sqrt{11}$	10	$\sqrt{11}+\sqrt{10}$	9
RCL	11	11	10	10	9	9
$\sqrt{x}$	$\sqrt{11}$	11	$\sqrt{10}$	10	$\sqrt{9}$	9
+	$\sqrt{11}$	11	$\sqrt{11}+\sqrt{10}$	10	$\sqrt{11}+\sqrt{10}+\sqrt{9}$	9
$x \leftrightarrow M$	11	$\sqrt{11}$	10	$\sqrt{11}+\sqrt{10}$	9	$\sqrt{11}+\sqrt{10}+\sqrt{9}$
−	11	$\sqrt{11}$	10	$\sqrt{11}+\sqrt{10}$	9	$\sqrt{11}+\sqrt{10}+\sqrt{9}$
1	1	$\sqrt{11}$	1	$\sqrt{11}+\sqrt{10}$	1	$\sqrt{11}+\sqrt{10}+\sqrt{9}$
=	10	$\sqrt{11}$	9	$\sqrt{11}+\sqrt{10}$	8	$\sqrt{11}+\sqrt{10}+\sqrt{9}$

It is essential to keep track of the memory in this way.

When a possible sequence is found, compile the program.

PROGRAM

LRN

00	x→M	08	=
01	+	09	SKZ
02	RCL	10	GOTO
03	$\sqrt{x}$	11	00
04	=	12	RCL
05	x→M	13	STOP
06	−	14	GOTO
07	1	15	00

LRN

EXECUTE

GOTO 00
Enter n
RUN ⟶ Σ
0 STO
Enter n
RUN ⟶ Σ
0 STO
.
.
.

Then execute

GOTO 00
Enter 5
RUN ⟶ 8.3823323
Enter 11
RUN ⟶ 25.784903
Enter 19
RUN ⟶ 57.193842
Enter 104
RUN ⟶ 711.95926

Practice Problem: Find

$$\sum_{i=1}^{n} \ln(i!) \quad \text{for } n = 5, 9, 24, 55$$

Answer: 10.450452 48.961295
559.68849 4010.7111

The [INT] Key

The [INT] key is used in programs 12 and 16 of Appendix A. You will use it to solve the next two problems.

Practice Problems: Write a program that rounds any decimal to the nearest cent. If you enter 53.7152 you should get 53.72. Similarly, if you enter 4.174, you should get 4.17.

Write a program to find the greatest perfect square less than or equal to any given number. (A perfect square is a number whose square root is an integer.)

Writing a Program

We have seen there are six steps in writing a program

(1) Write out the formula.
(2) Rearrange if necessary.
(3) Write out keysequence for formula.
(4) Using a table to keep track of the memory, develop key sequence for memory operators.
(5) Use different techniques to shorten program if necessary.
(6) Compile, run, debug.

We have seen how step (2) has been used to compute polynomials. There is another trick you can use to compute powers. Suppose you wish to use

$$9.7^x$$

in a computation. This must be rearranged using the formula

$$y^x = e^{\ln y^x} = e^{x \ln y}$$

Thus,

$$9.7^x = e^{x \ln 9.7}$$

This new arrangement is compatible with the machine.

It is important to use a table to keep track of the memory (step 4) when creating a program. For an example of this technique, see page 29.

If your program turns out to be too large, don't give up. You may be able to rearrange your formula to make the program shorter. If this does not work, part of the program may be entered manually during the execution of the program. For example, the program for the quadratic formula (program 7, appendix A) is too long. The last part must be entered manually: – RCL – RCL = .

Finally, you compile the program, run a test problem and get the wrong answer. You must debug the program.

Use the [SSTP] key to run the program step-by-step. You will thus find where the program goes wrong.

The biggest cause of errors is the omission of the [=] key. For example,

3 + 4 $\sqrt{x}$ =

computes $3 + \sqrt{4}$. If, instead, you wanted $\sqrt{3 + 4}$, you have misplaced the = sign:

3 + 4 = $\sqrt{x}$

Check your program for ln, e^x, $\sqrt{x}$, x^2, sin etc. An equals sign before these operators makes a big difference.

Appendices

Appendix A. Useful Programs

1. y^x

This program calculates y^x using the formula

$$y^x = e^{x \ln y} \qquad y > 0$$

PROGRAM		EXECUTE
LRN		GOTO 00
00 ln	04 e^x	Enter y
01 ×	05 STOP	RUN
02 STOP	06 GOTO	Enter x
03 =	07 00	RUN ⟶ y^x
LRN		

2. $\sqrt[x]{y}$

This program calculates $\sqrt[x]{y}$ using the formula

$$\sqrt[x]{y} = e^{\ln y \div x} \qquad y > 0$$

PROGRAM		EXECUTE
LRN		GOTO 00
00 ln	04 e^x	Enter y
01 ÷	05 STOP	RUN
02 STOP	06 GOTO	Enter x
03 =	07 00	RUN ⟶ $\sqrt[x]{y}$
LRN		

3. **Fibonacci Sequence**

This program computes the Fibonacci sequence

$$x_1, x_2, x_3, \cdots$$

using the formula

$$x_1 = 0 \quad x_2 = 1$$
$$x_i = x_{i-2} + x_{i-1} \qquad i = 3, 4, 5, \cdots$$

PROGRAM		EXECUTE
LRN		GOTO 00
00 0	05 x→M	RUN ⟶ x_1
01 STOP	06 STOP	RUN ⟶ x_2
02 1	07 M+	RUN ⟶ x_3
03 STO	08 GOTO	· ·
04 STOP	09 05	· ·
LRN		· ·

4. **Base 10 → Base 2**

This program converts a base 10 number n_{10} to a base 2 number x_2. Denote the digits of x_2 by

$$d_k\, d_{k-1}\, d_{k-2} \cdots d_1\, d_0$$

To execute the program, choose a value for k bigger than the expected number of digits in x_2. For example, to change 5964 to binary form choose k = 13 (2^{13} = 8192).

PROGRAM		EXECUTE
LRN		GOTO 00
00 STO	12 GOTO	Enter n_{10}
01 2	13 18	RUN
02 ln	14 0	Enter k
03 ×	15 STOP	RUN → d_k
04 STOP	16 GOTO	RUN
05 =	17 01	Enter k − 1
06 e^x	18 STO	RUN → d_{k-1}
07 −	19 1	RUN
08 RCL	20 STOP	Enter k − 2
09 x↔y	21 GOTO	RUN → d_{k-2}
10 =	22 01	⋮
11 SKN		⋮
LRN		RUN
		Enter 1
		RUN → d_1

$$d_0 = \begin{cases} 0 \text{ if } n_{10} \text{ is even} \\ 1 \text{ if } n_{10} \text{ is odd} \end{cases}$$

Note: You can modify this program to convert numbers to a base different from 2. Step 01 should be changed to this new base. For each digit d_i, repeat the sequence

RUN
Enter i
RUN → 0 or 1

until a 0 appears. The number of 1s generated by this procedure is the d_i digit.

5. **Base n → Base 10**

This program converts a base n number x_n to a base 10 number y_{10}. Denote the digits of x_n by

$$d_k\, d_{k-1}\, d_{k-2} \ldots d_1\, d_0$$

PROGRAM		EXECUTE
LRN		GOTO 00
00 STO	04 x	Enter n
01 0	05 RCL	RUN
02 +	06 GOTO	Enter d_k
03 STOP	07 02	RUN
LRN		Enter d_{k-1}
		RUN
		.
		.
		.
		Enter d_1
		RUN
		Enter d_0
		= ——→ y_{10}

6. **Additional Memory**

You can use the programming registers to store frequently-used constants. Suppose you will be making numerous calculations with the constants

$C = 2.998 \times 10^8$
$G = 6.673 \times 10^{-11}$
$K = 172$

These can be entered into the program and whenever one of these values appears in a computation, press GOTO and the appropriate line number. The desired constant appears on the display and you may continue with the calculations.

PROGRAM		EXECUTE
LRN		GOTO 00
00 2	11 7	RUN ——→ C
01 .	12 3	or
02 9	13 EE	GOTO 08
03 9	14 1	RUN ——→ G
04 8	15 1	or
05 EE	16 +/–	GOTO 18
06 8	17 STOP	RUN ——→ K
07 STOP	18 1	
08 6	19 7	
09 .	20 2	
10 6	21 STOP	
LRN		

7. **Quadratic Formula**

This program computes the solution of

$$ax^2 + bx + c = 0$$

using the formula

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

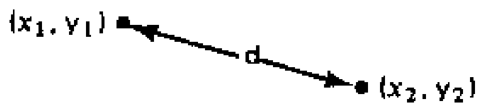
$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

If the roots are complex numbers, an error will occur.

PROGRAM		EXECUTE
LRN		GOTO 00
00 ÷	12 x↔M	Enter c
01 STOP	13 −	RUN
02 STO	14 RCL	Enter a
03 =	15 x^2	RUN
04 x↔M	16 =	Enter b
05 x	17 +/−	RUN ⟶ x_1
06 2	18 =	− RCL − RCL
07 ÷	19 $\sqrt{x}$	= ⟶ x_2
08 STOP	20 +	
09 =	21 x→M	
10 +/−	22 =	
11 1/x	23 STOP	
LRN		

8. **Distance Between (x_1, y_1) and (x_2, y_2)**

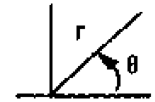
This program computes the distance between two points on the Cartesian plane.

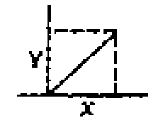


$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

PROGRAM		EXECUTE
LRN		GOTO 00
00 −	08 =	Enter x_1
01 STOP	09 x^2	RUN
02 =	10 +	Enter x_2
03 x^2	11 RCL	RUN
04 STO	12 =	Enter y_1
05 STOP	13 $\sqrt{x}$	RUN
06 −	14 STOP	Enter y_2
07 STOP		RUN ⟶ d
LRN		

9. **Polar → Rectangular Coordinates**





Polar Coordinates (r, θ) **Rectangular Coordinates** (x, y)

This program converts (r, θ) to (x, y) using the formulae

$x = r\cos\theta$
$y = r\sin\theta$

Step 2 of the execution is used to enter the angle mode of θ. This step may be left out in successive computations unless the angle mode is to be changed.

PROGRAM		EXECUTE
LRN		GOTO 00
00 x	07 RCL	deg, rad or grad
01 STOP	08 tan	Enter r
02 STO	09 =	RUN
03 cos	10 STOP	Enter θ
04 =	11 GOTO	RUN ⟶ x
05 STOP	12 00	RUN ⟶ y
06 x		
LRN		

10. **Rectangular → Polar Coordinates**

This program converts (x, y) to (r, θ) using

$r = x \div \cos\theta$

$$\theta = \begin{cases} \arctan\frac{y}{x} \text{ if } x > 0 \\ \arctan\frac{y}{x} + 180° \text{ if } x < 0 \end{cases}$$

This program will not compute (r, θ) if $x = 0$. Before entering the program, you must decide which angle mode you want θ to be expressed in and adjust steps 00, 10, 13–15 accordingly.

PROGRAM		EXECUTE
LRN		GOTO 00
00 deg (rad, grad)	12 +	Enter x
01 STO	13 (180	RUN
02 ÷	14 π or	Enter y
03 STOP	15 200)	RUN → θ
04 x↔y	16 =	RUN → r
05 =	17 x↔M	
06 arctan	18 ÷	
07 x↔M	19 RCL	
08 SKN	20 STOP	
09 GOTO	21 cos	
10 18(16)	22 =	
11 x↔M	23 STOP	
LRN		

11. d/m/s → degrees

This program converts a degrees/minutes/seconds value $x°y'z''$ to a decimal degrees value w°. This can also be used for hours/minutes/seconds conversion to decimal hours. The formula is

$$w = x + \frac{y}{60} + \frac{z}{3600}$$

PROGRAM		EXECUTE
LRN		GOTO 00
00 STO	10 6	Enter x
01 STOP	11 0	RUN
02 ÷	12 0	Enter y
03 6	13 =	RUN
04 0	14 M+	Enter z
05 =	15 RCL	RUN → w
06 M+	16 STOP	
07 STOP	17 GOTO	
08 ÷	18 00	
09 3		
LRN		

12. degrees → d/m/s

This program converts a decimal degrees value w° to a degrees/minutes/seconds value $x°y'z''$. This can also be used to convert decimal hours to hours/minutes/seconds. The formulae are

x = Int (w)
y = Int (60 x (w – x))
z = 60 x [60 x (w – x) – y]

PROGRAM		EXECUTE
LRN		GOTO 00
00 STO	12 INT	Enter w
01 INT	13 STOP	RUN → x
02 STOP	14 –	RUN → y
03 –	15 RCL	RUN → z
04 RCL	16 x↔y	
05 x↔y	17 x	
06 =	18 6	
07 x	19 0	
08 6	20 =	
09 0	21 STOP	
10 =	22 GOTO	
11 STO	23 00	
LRN		

13. Compound Interest

Let P = principal
i = interest rate compounded k times a year (expressed as a decimal)
B = balance after n compoundings

The formula for compound interest is

$$B = P\left(1 + \frac{i}{k}\right)^n$$

Program A: This program computes B given P, i, n, k.

PROGRAM		EXECUTE
LRN		GOTO 00
00 ÷	08 =	Enter i
01 STOP	09 e^x	RUN
02 +	10 x	Enter k
03 1	11 STOP	RUN
04 =	12 =	Enter n
05 ln	13 STOP	RUN
06 x	14 GOTO	Enter P
07 STOP	15 00	RUN ⟶ B
LRN		

Program B: This program computes P given B, i, n, k.

PROGRAM		EXECUTE
LRN		GOTO 00
00 ÷	09 e^x	Enter i
01 STOP	10 ÷	RUN
02 +	11 STOP	Enter k
03 1	12 x↔y	RUN
04 =	13 =	Enter n
05 ln	14 STOP	RUN
06 x	15 GOTO	Enter B
07 STOP	16 00	RUN ⟶ P
08 =		
LRN		

Program C: This program computes n given B, P, i, k.

PROGRAM		EXECUTE
LRN		GOTO 00
00 ÷	10 =	Enter B
01 STOP	11 ln	RUN
02 =	12 ÷	Enter P
03 ln	13 RCL	RUN
04 STO	14 x↔y	Enter i
05 STOP	15 =	RUN
06 ÷	16 STOP	Enter k
07 STOP	17 GOTO	RUN ⟶ n
08 +	18 00	
09 1		
LRN		

Program D: This program computes i given B, P, n, k

PROGRAM		EXECUTE
LRN		GOTO 00
00 ÷	08 −	Enter B
01 STOP	09 1	RUN
02 =	10 x	Enter P
03 ln	11 STOP	RUN
04 ÷	12 =	Enter n
05 STOP	13 STOP	RUN
06 =	14 GOTO	Enter k
07 e^x	15 00	RUN ⟶ i
LRN		

14. **Loans**

Define P = principal
PMT = payment amount
n = number of payments
i = interest rate (expressed as a decimal)
k = number of payments in 1 year

The formula for loans is

$$P = PMT \left[\frac{1 - \left(1 + \frac{i}{k}\right)^{-n}}{\frac{i}{k}} \right]$$

Program A: This program computes P given PMT, i, n, k.

PROGRAM		EXECUTE
LRN		GOTO 00
00 ÷	12 e^x	Enter i
01 STOP	13 –	RUN
02 =	14 1	Enter k
03 STO	15 $x \leftrightarrow y$	RUN
04 +	16 ÷	Enter n
05 1	17 RCL	RUN
06 =	18 x	Enter PMT
07 In	19 STOP	RUN ⟶ P
08 x	20 =	
09 STOP	21 STOP	
10 +/–	22 GOTO	
11 =	23 00	
LRN		

Program B: This program computes PMT given P, i, n, k.

PROGRAM		EXECUTE
LRN		GOTO 00
00 ÷	12 e^x	Enter i
01 STOP	13 –	RUN
02 =	14 1	Enter k
03 STO	15 $x \leftrightarrow y$	RUN
04 +	16 ÷	Enter n
05 1	17 RCL	RUN
06 =	18 ÷	Enter P
07 In	19 STOP	RUN ⟶ PMT
08 x	20 $x \leftrightarrow y$	
09 STOP	21 =	
10 +/–	22 STOP	
11 =		
LRN		

Program C: This program computes n given P, PMT, i, k.

PROGRAM		EXECUTE
LRN		GOTO 00
00 ÷	12 STOP	Enter i
01 STOP	13 –	RUN
02 =	14 1	Enter k
03 STO	15 $x \leftrightarrow y$	RUN
04 +	16 =	Enter P
05 1	17 In	RUN
06 =	18 ÷	Enter PMT
07 In	19 RCL	RUN ⟶ n
08 $x \leftrightarrow M$	20 =	
09 x	21 +/–	
10 STOP	22 STOP	
11 ÷		
LRN		

15. **Periodic Savings**

Define PMT = amount deposited k times a year at equal intervals
i = interest rate expressed as a decimal
n = number of deposits
FV = total value of the account at the end of the term.

The formula is

$$FV = PMT \times \left(1 + \frac{i}{k}\right) \times \left[\frac{\left(1 + \frac{i}{k}\right)^n - 1}{i/k}\right]$$

Program A: This program computes FV given PMT, i, n, k.

PROGRAM		EXECUTE
LRN		GOTO 00
00 +	12 RCL	Enter i [÷] k
01 1	13 =	RUN
02 =	14 $x \leftrightarrow M$	Enter n
03 STO	15 –	RUN
04 In	16 1	Enter PMT
05 x	17 ÷	RUN ⟶ FV
06 STOP	18 RCL	

07 =	19 $x \leftrightarrow y$
08 e^x	20 x
09 −	21 STOP
10 1	22 =
11 x	23 STOP
LRN	

Program B: This program computes PMT given FV, i, n, k.

PROGRAM		EXECUTE
LRN		GOTO 00
00 +	12 RCL	Enter i [÷] k
01 1	13 =	RUN
02 =	14 $x \leftrightarrow M$	Enter n
03 STO	15 −	RUN
04 ln	16 1	Enter FV
05 x	17 ÷	RUN ⟶ PMT
06 STOP	18 RCL	
07 =	19 x	
08 e^x	20 STOP	
09 −	21 =	
10 1	22 STOP	
11 x		
LRN		

Program C: This program computes n given FV, PMT, i, k.

PROGRAM		EXECUTE
LRN		GOTO 00
00 ÷	12 ÷	Enter i
01 STOP	13 STOP	RUN
02 =	14 +	Enter k
03 STO	15 1	RUN
04 +	16 =	Enter FV
05 1	17 ln	RUN
06 =	18 ÷	Enter PMT
07 $x \leftrightarrow M$	19 RCL	RUN ⟶ n
08 x	20 ln	
09 STOP	21 =	
10 ÷	22 STOP	
11 RCL		
LRN		

Program 16: Dice

This program simulates the roll of a die. Choose a 4 digit decimal n, to begin the sequence. For example, n = .3951

PROGRAM		EXECUTE
LRN		GOTO 00
00 RCL	12 =	Enter n STO
01 +	13 STO	RUN ⟶ die
02 π	14 INT	RUN ⟶ die
03 =	15 −	RUN ⟶ die
04 x^2	16 6	.
05 STO	17 =	.
06 −	18 SKN	.
07 RCL	19 GOTO	
08 INT	20 00	
09 x	21 STOP	
10 1	22 GOTO	
11 0	23 00	
LRN		

Appendix B. Mathematical Formulae

1. General

Quadratic

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$\text{where } \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Distance between (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Exponential and Logarithmic Identities

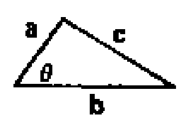
$a^0 = 1$ $\qquad (a^x)(a^y) = a^{x+y}$ $\qquad \ln ab = \ln a + \ln b$

$\frac{1}{a^x} = a^{-x}$ $\qquad a^x / a^y = a^{x-y}$ $\qquad \ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$(ab)^x = a^x b^x$ $\qquad (a^x)^y = a^{xy}$ $\qquad \ln(y^x) = x \ln y$

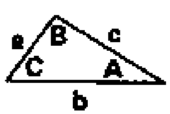
$$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$$

Law of Cosines



$$a^2 + b^2 - 2\,ab\cos\theta = c^2$$

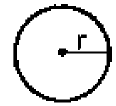
Law of Sines



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

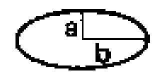
2. Geometry

Circle

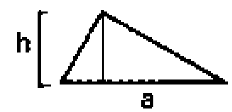


Sphere

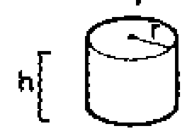
Ellipse



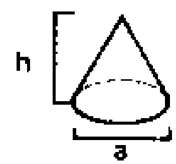
Triangle



Cylinder



Cone



Circumference

Circle	$2\pi r$

Area

Circle	πr^2
Sphere	$4\pi r^2$
Ellipse	πab
Triangle	$1/2\ ab$

Volume

Sphere	$4/3\pi r^3$
Cylinder	$\pi r^2 h$
Cone	$\frac{\pi a^2 h}{12}$

Equation

Circle	$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
Parabola	$y^2 = \pm\, 2\,px$
Line	$y = mx + b$

Appendix B. Mathematical Formulae (cont)

3–DERIVATIVES

General

$\frac{d(c)}{dx} = 0$

$\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$\frac{d(x^n)}{dx} = nx^{n-1}$

$\frac{d(u \cdot v)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{d(cu)}{dx} = c\frac{du}{dx}$

$\frac{d(u/v)}{dx} = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2}$

(Chain Rule) $\frac{dz}{dx} = \frac{dz}{dy} \times \frac{dy}{dx}$

Trigonometric

$\frac{d(\sin x)}{dx} = \cos x$

$\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\frac{d(\cos x)}{dx} = -\sin x$

$\frac{d(\cos^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\frac{d(\tan x)}{dx} = \sec^2 x$

$\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$

Hyperbolic

$\frac{d(\cosh x)}{dx} = \sinh x$

$\frac{d(\cosh^{-1}x)}{dx} = \frac{1}{\sqrt{x^2-1}}$

$\frac{d(\sinh x)}{dx} = \cosh x$

$\frac{d(\sinh^{-1}x)}{dx} = \frac{1}{\sqrt{x^2+1}}$

$\frac{d(\tanh x)}{dx} = \text{sech}^2 x$

$\frac{d(\tanh^{-1}x)}{dx} = \frac{1}{1-x^2}$

Transcendental

$\frac{d(\ln x)}{dx} = \frac{1}{x}$

$\frac{d(e^x)}{dx} = e^x$

$\frac{d(a^x)}{dx} = a^x \ln a$

$\frac{d(u^v)}{dx} = vu^{v-1} \cdot \frac{du}{dx} + \ln u \cdot u^v \cdot \frac{dv}{dx}$

4–INTEGRALS

$\int du = u + C$

$\int a\, du = au + C$ where a is any constant

$\int [f(u) + g(u)]\, du = \int f(u)\, du + \int g(u)\, du$

$\int u^n\, du = \frac{u^{n+1}}{n+1} + C \quad n \neq -1$

$\int \frac{du}{u} = \ln |u| + C$

$\int a^u\, du = \frac{a^u}{\ln a} + C$

$\int e^u\, du = e^u + C$

$\int \sin u\, du = -\cos u + C$

$\int \cos u\, du = \sin u + C$

$\int \sec^2 u\, du = \tan u + C$

$\int \csc^2 u\, du = -\cot u + C$

$\int \sec u \tan u\, du = \sec u + C$

$\int \csc u \cot u\, du = -\csc u + C$

$\int \tan u\, du = \ln |\sec u| + C$

$\int \cot u\, du = \ln |\sin u| + C$

$\int \sec u\, du = \ln |\sec u + \tan u| + C$

$\int \csc u\, du = \ln |\csc u - \cot u| + C$

$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$ where $a > 0$

$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$

$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \left|\frac{u}{a}\right| + C$ where $a > 0$

$\int \sinh u\, du = \cosh u + C$

$\int \cosh u\, du = \sinh u + C$

$\int \text{sech}^2 u\, du = \tanh u + C$

$\int \text{csch}^2 u\, du = -\coth u + C$

$\int \text{sech}\, u \tanh u\, du = -\text{sech}\, u + C$

$\int \text{csch}\, u \coth u\, du = -\text{csch}\, u + C$

Integration by parts

$\int u\, dv = uv - \int v\, du$

Appendix C. Physics Concepts

1–PHYSICAL CONSTANTS

Name of Quantity	Symbol	Value
Speed of light in vacuum	c	2.9979×10^{8} m s^{-1}
Charge of electron	q_e	-1.602×10^{-19} C
Rest mass of electron	m_e	9.10×10^{-31} kg
Ratio of charge to mass of electron	q_e/m_e	1.759×10^{11} C kg^{-1}
Planck's constant	h	6.626×10^{-34} J s
Boltzmann's constant	k	1.381×10^{-23} J K^{-1}
Avogadro's number (chemical scale)	N_0	6.023×10^{23} molecules $mole^{-1}$
Universal gas constant (chemical scale)	R	8.314 J $mole^{-1}$ K^{-1}
Mechanical equivalent of heat	J	4.185×10^{3} J $kcal^{-1}$
Standard atmospheric pressure	1 atm	1.013×10^{5} N m^{-2}
Volume of ideal gas at 0° C and 1 atm (chemical scale)		22.415 liters $mole^{-1}$
Absolute zero of temperature	0 K	–273.15° C
Acceleration due to gravity (sea level, at equator)		9.78049 m s^{-2}
Universal gravitational constant	G	6.673×10^{-11} N • m^2 kg^{-2}
Mass of earth	m_E	5.975×10^{24} kg
Mean radius of earth		6.371×10^{6} m = 3959 mi
Equatorial radius of earth		6.378×10^{6} m = 3963 mi
Mean distance from earth to sun	1 AU	1.49×10^{11} m = 9.29×10^{7} mi
Eccentricity of earth's orbit		0.0167
Mean distance from earth to moon		3.84×10^{8} m = 60 earth radii
Diameter of sun		1.39×10^{9} m = 8.64×10^{5} mi
Mass of sun	m_s	1.99×10^{30} kg = 333,000 x mass of earth
Coulomb's law constant	$k = 1/4\pi\epsilon_0$	8.9874×10^{9} N • m^2 C^{-2}
Faraday's constant (1 faraday)	F	96.487 C $mole^{-1}$
Mass of neutral hydrogen atom	m_H^1	1.007825 amu
Mass of proton	m_p	1.007277 amu
Mass of neutron	m_n	1.008665 amu
Mass of electron	m_e	5.486×10^{-4} amu
Ratio of mass of proton to mass of electron	m_p/m_e	1836.11
Rydberg constant for nucleus of infinite mass	R_∞	109.737 cm^{-1}
Rydberg constant for hydrogen	R_H	109.678 cm^{-1}
Wien displacement law constant		0.2898 cm K^{-1}

Appendix C. Physics Concepts (cont)

2–CONVERSIONS

English to Metric

To Find	Multiply	By
microns	mils	**25.4**
centimeters	inches	**2.54**
meters	feet	**0.3048**
meters	yards	**0.9144**
kilometers	miles	**1.609344**
grams	ounces	28.349523
kilograms	pounds	**0.45359237**
liters	gallons(U.S.)	3.7854118
liters	gallons(Imp.)	4.546090
milliliters(cc)	fl. ounces	29.573530
sq. centimeters	sq. inches	**6.4516**
sq. meters	sq. feet	**0.09290304**
sq. meters	sq. yards	**0.83612736**
milliliters(cc)	cu. inches	**16.387064**
cu. meters	cu. feet	2.8316847×10^{-2}
cu. meters	cu. yards	0.76455486

Temperature Conversions

$$F = \frac{9}{5}(C) + 32$$

$$C = \frac{5}{9}(F - 32)$$

General

To Find	Multiply	By
atmospheres	feet of water @ 4°C	.0294990
atmospheres	inches of mercury @ 0°C	.0334211
atmospheres	pounds per sq. inch	.068046
BTU	foot-pounds	.00128593
BTU	joules	9.4845×10^{-4}
cu. ft.	cords	**128**
ergs	foot-pounds	13558200
feet	miles	**5280**
feet of water @ 4°C	atmosphere	33.8995
foot-pounds	horsepower-hours	1.98×10^{6}
foot-pounds	kilowatt-hours	2655220
foot-pounds per min.	horsepower	3.3×10^{4}
horsepower	foot-pounds per sec.	.00181818
inches of mercury @ 0°C	pounds per sq. inch	2.03602
joules	BTU	1054.3504
joules	foot-pounds	1.35582
kilowatts	BTU per min.	.01757251
kilowatts	foot-pounds per min.	2.2597×10^{-5}
kilowatts	horsepower	.7457
knots	miles per hour	0.86897624
miles	feet	1.89393×10^{-4}
nautical miles	miles	0.86897624
sq. feet	acres	**43560**
watts	BTU per min.	17.5725

Boldface numbers are exact; others are rounded.

Appendix C. Physics Concepts (cont)

3—TABLE OF UNITS

Supplementary Tables Units for a System of Measures for International Relations		
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	Kelvin	K
Luminous intensity	candela	cd

Prefix Names of Multiples and Submultiples of Units

Factor by which unit is multiplied	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Derived Units			
Acceleration	m/s^2		meter per second squared
Activity (of radioactive source)	s^{-1}		1 per second
Angular acceleration	rad/s^2		radian per second squared
Angular velocity	rad/s		radian per second
Area	m^2		square meter
Density	kg/m^3		kilogram per cubic meter
Dynamic viscosity	$N \cdot s/m^2$		newton-second per sq meter
Electric capacitance	F	$(A \cdot s/V)$	farad
Electric charge	C	$(A \cdot s)$	coulomb
Electric field strength	V/m		volt per meter
Electric resistance		(V/A)	ohm
Entropy	J/K		joule per kelvin
Force	N	$(kg \cdot m/s^2)$	newton
Frequency	Hz	(s^{-1})	hertz
Illumination	lx	(lm/m^2)	lux
Inductance	H	$(V \cdot s/A)$	henry
Kinematic viscosity	m^2/s		sq meter per second
Luminance	cd/m^2		candela per sq meter
Luminous flux	lm	$(cd \cdot sr)$	lumen
Magnetomotive force	A		ampere
Magnetic field strength	A/m		ampere per meter
Magentic flux	Wb	$(V \cdot s)$	weber
Magentic flux density	T	(Wb/m^2)	tesla
Power	W	(J/s)	watt
Pressure	N/m^2		newton per square meter
Radiant intensity	W/sr		watt per steradian
Specific heat	J/kg K		joule per kilogram kelvin
Thermal conductivity	W/m K		watt per meter kelvin
Velocity	m/s		meter per second
Volume	m^3		cubic meter
Voltage, Potential difference, Electromotive force	V	(W/A)	volt
Wave number	m^{-1}		1 per meter
Work, energy, quantity of heat	J	$(N \cdot m)$	joule

Appendix D.
Batteries and Maintenance

AC Operation

If you have bought or own a Commodore adapter, connect this optional adapter to any standard electrical outlet and plug the jack into the calculator. After the above connections have been made, the power switch may be turned "ON." (While connected to AC, the battery may be left in place or removed but we recommend removal.

Use proper Commodore/CBM adapter for AC operation. Adapter 640 or 707 North America; Adapter 708 England; Adapter 709 West Germany.

Battery Operation

Push the power switch "ON." An interlock switch in the calculator socket will prevent battery operation if the adapter jack remains connected.

Your new calculator uses one ordinary 9 volt rectangular battery, available virtually anywhere. The connector must be attached firmly to the two battery terminals.

Low Power

If battery is low, calculator display:

a. will appear erratic
b. will dim
c. will fail to accept numbers

If one or all of the above conditions occur, you may check for a low battery condition by entering a series of 8's. If 8's fail to appear, operations should not be continued on battery power. Unit may be operated on AC power.

CAUTION

A strong static discharge will damage your machine.

Shipping Instructions

A defective machine should be returned to the authorized service center nearest you. See listing of service centers.

TEMPERATURE RANGE

Mode	Temperature °C	Temperature °F
Operating	0° to 50°	32° to 122°
Storage	-40° to 55°	-40° to 131°

For a copy of SOLUTIONS TO THE PRACTICE PROBLEMS IN THE P50 MANUAL, send $3.50 to cover cost of handling to the personal attention of Mr. Sam Bernstein, Nassau, Bahamas.